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Branes in Super–AdS Backgrounds and Superconformal Theories [1]

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Abstract

We demonstrate how actions for interacting superconformal field theories in (p+1)-dimensions arise as a result of gauge fixing worldvolume diffeomorphisms and fermionic κ-symmetry in actions for super-p-branes propagating in superbackgrounds of $AdS_{p+2} \times S^{D-p-2}$ geometry. The method of nonlinear realizations and coset spaces is used for getting an explicit form of supervielbeins and superconnections of the $AdS \times S$ superbackgrounds, which are required for the construction of the superconformal theories. Subtleties of consistent gauge fixing worldvolume symmetries of the branes are discussed.

During the last two years there have been a great interest and an intensive development of the Maldacena conjecture [1] which suggests that supergravity theories (or, more generally, superstring theories and M–theory) on spacetimes with a geometry of anti–de–Sitter times a compact manifold (for instance, a sphere S) are in a certain sense equivalent (or dual) to superconformal theories living on a boundary of the AdS space. More precise statement is that the generating functional of correlation functions of observables of the conformal field theory on the boundary is equal to the partition function of supergravity (or string) theory in the bulk [1]. In a classical approximation this reads

$$\langle \exp \int_{\mathcal{M}} \Phi_0 \mathcal{O} \rangle_{CFT} = \exp\left(-S_{AdS}(\Phi)\right), \qquad (1)$$

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where $S_{AdS}(\Phi)$ is the action for the fields Φ of the bulk theory, and Φ_0 are boundary values of Φ, which are considered as sources for conformal fields (operators) $\mathcal{O}$ of the conformal theory on $\mathcal{M}$.

For more profound analysis and check of this correspondence it is desirable to know the detailed structure of the bulk theory (for instance, IIB superstring in $AdS_5 \times S^5$) and the structure of the superconformal theory on the boundary. This is especially essential for understanding field interactions.

This information can be provided by corresponding effective actions invariant under superconformal transformations. A natural way of getting such actions is to consider the dynamics of superbranes in $AdS \times S$ backgrounds, from which, in fact, the AdS/CFT conjecture originated.

In this connection an extensive work has been undertaken by several theoretical groups to derive superconformal field theory actions from $d = p + 1$ worldvolume actions of superbranes in $AdS_{p+2} \times S^{D-p-2}$ backgrounds of D–dimensional supergravity [2]–[7].

The idea is rather simple and natural. The $AdS_{p+2} \times S^{D-p-2}$ superbackground has isometry symmetry described by a supergroup G. For instance, in the case of an M–theory membrane, when $p = 2$ and $D = 11$, G is an orthosimplectic supergroup $OSp(8|4)$; in the M5–brane case $p = 5, D = 11$ and $G = OSp(2, 6|4)$; and in the case of the IIB D3–brane $p = 3$, $D = 10$ and $G = SU(2, 2|4)$. (The numbers in the names of the supergroups correspond to their bosonic subgroups). The maximal bosonic subgroup of G (which is the bosonic isometry of $AdS_{p+2} \times S^{D-p-2}$) is $SO(2, p+1) \times SO(D-p-1)$, and the fermionic transformations of G are generated by 32 Grassmann generators $\hat{Q}_\alpha$. Now recall that $SO(2, p+1)$ is also a group of conformal transformations acting on a (p+1)–dimensional space–time $\mathcal{M}_{p+1}$ associated with the boundary of AdS_{p+2}. In the approach under consideration the role of $\mathcal{M}_{p+1}$ is played by the $d = p + 1$ worldvolume of the corresponding superbrane. The whole supergroup G will thus be associated with superconformal symmetry of a worldvolume field theory describing the dynamics of the physical modes of the superbrane.

Consider in detail how the superconformal theory is derived from the superbrane action with the use of the example of an M2–brane in $AdS_4 \times S^7$. All other more complicated cases (D3–brane, M5–brane etc.) can be treated in a similar way.

Let us start with the discussion of general properties of superbrane worldvolume actions. The action for a supermembrane propagating in a $D = 11$

supergravity background has the following form [8]

$$S_{M2} = -\int_{\mathcal{M}_3} d^3\xi \sqrt{-\det g_{ij}} - \int_{\mathcal{M}_3} d^3\xi \varepsilon^{ijk} \partial_i Z^L \partial_j Z^M \partial_k Z^N A_{NML}(Z), \qquad (2)$$

where $g_{ij}(\xi) = \partial_i Z^M E^a_M(Z) E_{aN}(Z) \partial_j Z^N$ $(i,j = 0,1,2;\ a = 0,1,...,10)$ is a worldvolume metric induced by embedding $\mathcal{M}_3$ (parametrized by ξ^i) into a curved $D = 11$ target superspace parametrized by bosonic coordinates X^m $(m = 0,1,...,10)$ and fermionic Majorana spinor coordinates Θ^α $(\alpha = 0,1,...,32)$ called all together Z^M.

$E^a = dZ^M E^a_M(Z)$ and $E^\alpha = dZ^M E^\alpha_M(Z)$ are supervielbeins describing the geometry of the target superspace. Their leading components correspond to the graviton $e^a_m(X) = E^a_m|_{\Theta=0}$ and the gravitino $\psi^\alpha_m(X) = E^\alpha_m|_{\Theta=0}$, and $A_{NML}(Z)$ is a three-form superfield whose leading component $A_{nml}(X) = A_{nml}(Z)|_{\Theta=0}$ is the gauge field of $D = 11$ supergravity.

The action (2) is invariant under target–space superdiffeomorphisms

$$Z'^M = Z'^M(Z), \qquad (3)$$

local worldvolume diffeomorphisms

$$\xi'^i = \xi'^i(\xi) \qquad (4)$$

and local fermionic κ–symmetry transformations

$$\delta_\kappa Z^M E^a_M = 0, \quad \delta_\kappa Z^M E^\alpha_M = \kappa^\beta(\xi)(1+\bar{\Gamma})_\beta{}^\alpha, \qquad (5)$$

where

$$\bar{\Gamma} = \frac{1}{6\sqrt{-g}} \varepsilon^{ijk} \Gamma_{ijk}, \qquad \bar{\Gamma}^2 \equiv 1 \qquad (6)$$

and hence $1+\bar{\Gamma}$ is a spinor projection matrix. Γ_{ijk} is an antisymmetric product of $D = 11$ gamma–matrices $(\Gamma_a)_\beta{}^\alpha$ pulled back on to the worldvolume, i.e $\Gamma_i \equiv \partial_i Z^M E^a_M \Gamma_a$.

The appearance of the spinor projector in the κ–transformations reflects the fact that the presence of the supermembrane in the target superspace breaks half the 32 supersymmetries of a $D = 11$ supergravity vacuum, the unbroken supersymmetries being associated with those Grassmann coordinates Θ^α which can be eliminated by κ–symmetry transformations, while remaining 16 Θ^α are worldvolume Goldstone fermions of the spontaneously broken supersymmetries and describe physical fermionic modes of supermembrane fluctuations.

An important requirement for the κ–transformations (5) to be a symmetry of the membrane action (2) is that the target–space supervielbeins $E^a(Z)$,

$E^\alpha(Z)$, superconnections $\Omega_b{}^a(Z)$ and the gauge superfield $A^{(3)}$ satisfy $D=11$ supergravity constraints. The most essential constraints are the torsion constraint

$$T^a = dE^a + E^b\Omega_b{}^a = iE^\alpha\Gamma^a_{\alpha\beta}E^\beta, \tag{7}$$

and the field–strength $F^{(4)} = dA^{(3)}$ constraint

$$F^{(4)} = \frac{i}{2}E^aE^bE^\alpha E^\beta(\Gamma_{ab})_{\alpha\beta} + \frac{1}{4!}E^aE^bE^cE^dF_{abcd}. \tag{8}$$

Other constraints are either conventional or can be obtained from (7) and (8) by considering their Bianchi identities.

The $D=11$ supergravity constraints are amount to supergravity equations of motion. Therefore, a supergravity background compatible with membrane κ–symmetry must be a solution of supergravity field equations. When the gravitino field is zero the supergravity equations are the Einstein equations for the $D=11$ curvature and Maxwell–like equations for $F^{(4)}$

$$R_{mn} - \frac{1}{2}g_{mn}R = \frac{1}{3}(F_{ml_1l_2l_3}F_n{}^{l_1l_2l_3} - \frac{1}{8}g_{mn}F^2);$$

$$D_pF^{plmn} = \frac{1}{576}\epsilon^{lmnl_1\ldots l_8}F_{l_1l_2l_3l_4}F_{l_5l_6l_7l_8}. \tag{9}$$

The $AdS_4\times S^7$ is one of the solutions of (9) found almost twenty years ago by Freund and Rubin [9] with the purpose to compactify $D=11$ supergravity ala Kaluza and Klein. For such a solution the gravitino field $\psi^\alpha_m(X)$ is zero.

It has been known that this solution is invariant under the maximum number of supersymmetry transformations whose 32 parameters satisfy an $AdS_4\times S^7$ Killing spinor condition.

As a metric on $AdS_4\times S^7$ it is convenient to take the following one

$$ds^2 = \left(\frac{r}{R}\right)^4 dx^i\eta_{ij}dx^j + \left(\frac{R}{r}\right)^2 dr^2 + R^2d\Omega^2\,, \tag{10}$$

where x^i, r $(i=0,1,2)$ are coordinates of the AdS_4 and $d\Omega^2$ stands for a metric of the sphere S^7 of a radius R parametrized by coordinates $y^{a'}$ $(a'=1',...,7')$. The coordinates x^i of AdS_4 will be identified with the worldvolume coordinates ξ^i upon imposing a static gauge.

When $r\to\infty$ the second term in (10) tends to zero and effectively the AdS part of the metric becomes three–dimensional. The flat $d=3$ metric $dx^i\eta_{ij}dx^j$ is associated with the AdS_4 boundary $\mathcal{M}_3$ which is a Minkowski space.

For this choice of the $AdS_4 \times S^7$ metric the gauge field $A^{(3)}(X) = A^{(3)}(Z)|_{\Theta=0}$ and its field strength $F^{(4)}$, which satisfy the $D=11$ supergravity equations, have the following components which are non–zero only in the AdS_4 part of space–time

$$A^{(3)} = dx^2 dx^1 dx^0 (\frac{r}{R})^6, \quad F^{(4)} = -6 dr dx^2 dx^1 dx^0 (\frac{r^5}{R^6}) \tag{11}$$

When the supermembrane propagates in the $AdS_4 \times S^7$ supergravity background its action (2) is invariant under the supergroup $OSp(8|4)$ of the isometries of this background, to which the target–space superdiffeomorphisms (3) are reduced. A superconformal form of the $osp(8|4)$ superalgebra will be presented a bit later.

At this stage $OSp(8|4)$ is not yet the superconformal group which acts on the worldvolume $\mathcal{M}_3$ of the membrane but it is rather an internal symmetry of the fields living on $\mathcal{M}_3$. For the $OSp(8|4)$ supergroup to become a worldvolume superconformal symmetry one should (in an appropriate way) gauge fix the local worldvolume diffeomorphisms (4) and the κ–symmetry (5) of the membrane action. This gauge fixing eliminates the pure–gauge degrees of freedom so that only fields which correspond to the physical modes of the superbrane remain in the theory.

An appropriate condition for fixing the worldvolume diffeomorphisms is a static gauge when worldvolume coordinates are identified with three coordinates of AdS_4

$$\xi^i = x^i \quad (i = 0, 1, 2) \tag{12}$$

(which at $r \to \infty$ parametrize the AdS_4 boundary). Thus the membrane worldvolume is associated with the AdS_4 boundary. In the static gauge eight physical bosonic worldvolume fields are the AdS radial coordinate $r(\xi)$ and the S^7 coordinates $y^{a'}(\xi)$.

A possible gauge fixing of κ–symmetry (5) compatible with the static gauge (12) is putting to zero the following 16 components of Θ^α

$$\eta^\alpha \equiv [(1 + \Gamma^{012})\Theta]^\alpha = 0. \tag{13}$$

Then other 16 coordinates

$$\theta^\alpha \equiv [(1 - \Gamma^{012})\Theta]^\alpha \tag{14}$$

remain as the fermionic physical fields on $\mathcal{M}_3$. Note that in the gauge (12) $1 + \Gamma^{012}$ coincides with the κ–symmetry projector $1 + \bar{\Gamma}$ when the membrane is in a static (vacuum) state, i.e. when the bosonic fields (r, y^i) are worldvolume constants and θ^α are zero. This makes the κ–symmetry gauge (13) admissible.

A combination of the worldvolume bosonic (4) and fermionic (5) transformations accompanied by the target superspace isometry transformations of $OSp(8|4)$ which preserves the gauge fixing conditions (12) and (13) now becomes a non–linearly realized superconformal symmetry on the supermembrane worldvolume. This is because we have identified the worldvolume coordinates with target space AdS coordinates, and the parameters of the worldvolume symmetries are now expressed in terms of the constant parameters of target superspace symmetry $OSp(8|4)$. For instance, the static gauge (12) is preserved if the parameters of the worldvolume transformations δ_w and κ-transformations δ_κ are related to the parameters of $OSp(8|4)$ transformations δ_{osp} as follows

$$0 = x^i - \xi^i = x'^i - \xi'^i = x^i - \xi^i + \delta_{osp}x^i + \delta_w x^i + \delta_\kappa x^i - \delta_w \xi^i \quad \Rightarrow$$

$$\delta_w x^i - \delta_w \xi^i + \delta_\kappa x^i = -\delta_{osp} x^i. \tag{15}$$

To find an explicit form of the superconformal variations of the physical fields $r(\xi)$, $y^{a'}(\xi)$ and $\theta^\alpha(\xi)$ and to derive from the original superbrane action the superconformal action describing the dynamics of these fields one should know an explicit form of the supervielbeins $E^a(Z)$, $E^\alpha(Z)$ and of the three–form superfield $A^{(3)}(Z)$ which enter the supermembrane action. This is a key point in the construction of the superconformal action.

At the moment we know only the leading components of these objects at $\Theta = 0$. These are the bosonic $AdS \times S$ metric (10) and the bosonic value of the $A^{(3)}(x, r)$ field (11).

A direct way of getting $E^a(Z)$, $E^\alpha(Z)$ and $A^{(3)}(Z)$ is to solve the supergravity constraints (7) and (8) taking the values (10) and (11) of the superforms at $\Theta = 0$ as initial conditions. This has been done by Claus [5]. However, this explicit form is rather complicated since $E^a(X, \Theta)$ and $E^\alpha(X, \Theta)$ are polynomials of up to the 32-nd power in Θ. Even if half of Θ are eliminated by gauge fixing κ–symmetry, this will, in general, result in polynomials of the 16-th power. So, if written in terms of these polynomials the supermembrane action looks very cumbersome and untreatable. Hence it is desirable to find simpler form of the AdS superforms.

An elegant way of looking for this form is to consider the $AdS_4 \times S^7$ superspace as a so called coset superspace.

The method of coset spaces has been applied to the description of $AdS \times S$ superspaces in a number of papers [2, 3, 4, 6, 7] and is based on a classical work of Cartan on group manifolds, symmetric and homogeneous spaces. It is worth mentioning that the first physical model with global supersymmetry [10] and the first supergravity [11] was constructed by using this method.

Let us briefly sketch basic ideas of this method by the use of the example of the AdS superspace. As we have already discussed the $AdS_4 \times S^7$ superspace has the isometry symmetry described by the supergroup $OSp(8|4)$. This means that a given point $Z^M = (X^m, \Theta^\alpha)$ of this superspace can be connected with another point by an $OSp(8|4)$ transformation. A subgroup of $G = OSp(8|4)$ which leaves a given point Z^M intact is called the stability (or isotropy) subgroup of G. In our case the stability group is a bosonic group $H = SO(1,3) \times SO(7)$ which acts on a superspace tangent to $AdS_4 \times S^7$. The connection forms which define the parallel transport in the AdS superspace take their values in the algebra of H.

The space with these properties is called the coset space and is denoted as

$$K = G/H.$$

It is the space of the classes of equivalence of the points of G which are related by the H–transformations.

In our case

$$K = \frac{OSp(8|4)}{SO(1,3) \times SO(7)}.$$

Note that the bosonic subspaces AdS_4 and S^7 of this superspace are coset spaces themselves, actually, they are symmetric spaces

$$AdS_4 = \frac{SO(2,3)}{SO(1,3)}, \qquad S^7 = \frac{SO(8)}{SO(7)}.$$

Note also that the difference between the dimensions of the group G and H is equal to the (bosonic plus fermionic) dimension of the coset space

$$dim\ K = dim\ G - dim\ H.$$

Thus the generators of G, called coset generators K, which are not contained in the stability subgroup of H, are in one to one correspondence with the coset space coordinates Z^M and are associated with the boosts of the points of the coset superspace.

Now remember that an element of the group G can be represented as an exponent of the generators G_I of the group times parameters of the group transformations λ^I, considered as coordinates of the group manifold G

$$G(\lambda) = \exp{(\lambda^I G_I)}. \tag{16}$$

By analogy, we can identify a point $Z^M = (X^m, \Theta^\alpha)$ of the AdS superspace with a coset element $K(Z)$ realized as an exponent

$$K(Z) = \exp{(X^a P_a + \Theta^\alpha \hat{Q}_\alpha)}, \tag{17}$$

where $P_a = (P_i, P_r; P_{a'})$ $(i = 0, 1, 2,\ a' = 1, ..., 7)$ are bosonic boosts acting, respectively, on AdS_4 and S^7, and Q_α are supertranslations (supercharges).

To get the variation properties of the AdS superspace coordinates X^m, Θ^α under $OSp(8|4)$ one should act on $K(Z)$ by a supergroup element $G(\lambda)$

$$K'(Z'(\lambda, Z)) = G(\lambda) K(Z).$$

If we now use $K(Z)$ to construct a so called Cartan one–form $L = K^{-1}(Z) dK(Z)$, it is claimed that this form takes values in the algebra of G, and its components are supervielbeins and superconnections describing the geometry of the AdS superspace

$$L = K^{-1}(Z) dK(Z) = E^a P_a + E^\alpha \hat{Q}_\alpha + \Omega^{ab} M_{ab}. \tag{18}$$

The one–forms E^a, E^α corresponding to the boost generators $P_a, \hat{Q}_\alpha$ are supervielbeins (which we can use to construct the supermembrane action), while the one–forms Ω^{ab} are connections in the AdS superspace taking their values in the stability subalgebra $h = so(1,3) \times so(7)$ generated by M_{ab}. Note that the indices a and α correspond now to the vector and a spinor representation of the stability group, respectively.

To convince oneself that E^a, E^α and Ω^{ab} have indeed the properties of vielbeins and connections, consider their transformation properties under the action of the stability group H, which acts on $K(Z)$ from the right:

$$K'(Z) = K(Z) H(Z),$$

$$L' = K'^{-1} dK' = H^{-1} (K^{-1} dK) H + H^{-1} dH. \tag{19}$$

Comparing (18) with (19) we see that E^a, E^α transform homogeneously under H–transformations, while Ω^{ab} acquires an inhomogeneous contribution from $H^{-1} dH$ which takes values in the algebra of the generators M_{ab}. Thus, E^a, E^α and Ω^{ab} indeed have the correct transformation properties of supervielbeins and superconnections under the tangent superspace stability group.

Now, taking an exponential parametrization (17) of the coset element $K(Z)$ (which is in fact not unique and corresponds to a choice of a local supercoordinate system), substituting it into the expression for the Cartan form (18) and making use of the exact form of the superalgebra of the $OSp(8|4)$ generators one can derive an explicit form of the supervielbeins and superconnections. In general they will be again polynomials of the 32-nd power in Θ, as has been obtained by Metsaev and Tseytlin [2]. And further work is required to check whether this polynomial dependence can be reduced down to lower powers in Θ due to some matrix identities, which is *a priori* not obvious.

An alternative way is to find a suitable exponential parametrization of the coset element $K(Z)$ which would directly produce the supervielbeins and superconnections as short polynomials in Θ. Such a parametrization has been found in [6]. The idea has been to arrange the generators of $OSp(8|4)$ in such a way that their (anti) commutators take the explicit form of the superconformal algebra acting on a $d=3$ subspace $\mathcal{M}_3$ of AdS_4. So let us take the following generators as the coset superspace generators which appear in the exponent of $K(Z)$

Π_i $(i=0,1,2)$ – the boost (momenta) generators on M_3;

$P_r = D$ – the dilatation generator;

$P_{a'}$ $(a'=1,...,7)$ – boosts on the S^7 sphere;

$Q_\alpha = (1+\Gamma^{012})\hat{Q}_\alpha$ – ordinary supersymmetry transformations;

$S_\alpha = (1-\Gamma^{012})\hat{Q}_\alpha$ – special superconformal transformations.

To close the $OSp(8|4)$ superalgebra we must add to these generators the generators which correspond to the stability group $SO(1,3)\times SO(7)$. These consist of $SO(7)$ rotations $M_{a'b'}$, $SO(1,2)$ rotations M_{ij} plus boost generators $M_{ir} = \Pi_i - K_i$ which all together form the generators M_{ab} of the $SO(1,3)$–rotations, and the generators K_i of special conformal transformations of $\mathcal{M}_3$.

The $osp(8|4)$ superalgebra written in terms of these generators has a relatively simple form

$$[\Pi_i,\Pi_j]=0,\ [\Pi_i,Q_\alpha]=0,\ \{Q,Q\}\sim\Gamma^i\Pi_i,\ [D,\Pi_i]=\Pi_i,\ \{D,Q_\alpha\}=\frac{1}{2}Q_\alpha, \tag{20}$$

$$[K_i,K_j]=0,\quad [K_i,S_\alpha]=0,\quad \{S,S\}\sim-\Gamma^iK_i,\quad [D,K_i]=-K_i, \tag{21}$$

$$\{D,S_\alpha\}=-\frac{1}{2}S_\alpha,$$

$$[\Pi_i,S]\sim-\Gamma_iQ,\quad [K_i,Q]\sim\Gamma_iS,$$

$$\{Q_\alpha,S_\beta\}=h_{\alpha\beta}{}^AT_A,\quad [T_A,Q_\alpha]=t_{A\alpha}{}^\beta Q_\beta,\quad [T_A,S_\alpha]=g_{A\alpha}{}^\beta S_\beta, \tag{22}$$

where T_A stand for the generators D, M_{ij}, $P_{a'}$ and $M_{a'b'}$. To close the $osp(8|4)$ superalgebra one should add to (20)–(22) commutation relations of Π_i and K_i with T_A. But it is not necessary to know these commutators explicitly for the derivation of the Cartan superform.

We see that the commutation relations of Π_i and Q, and K_i and S form three–dimensional super Poincare subalgebras of $osp(8|4)$.

Now, as the exponent parametrization of the coset $K(Z)$ let us take

$$K = e^{x^i \Pi_i} e^{(\log \frac{r}{R})D} e^{y^{a'} P_{a'}} e^{\eta^\alpha Q_\alpha} e^{\theta^\alpha S_\alpha}, \tag{23}$$

where $\eta = (1 + \Gamma^{012})\Theta$ and $\theta = (1 - \Gamma^{012})\Theta$ are projected AdS Grassmann coordinates already considered above in connection with κ–symmetry gauge fixing (13).

Using this parametrization and the form (20)–(22) of the $osp(8|4)$ commutation relations we can compute the components of the Cartan form $K^{-1}dK$ which appear to be polynomials of only 6-th power in θ and η. We can simplify the form of the supervielbeins even more if we gauge fix the κ–symmetry of the supermembrane in a suitable way. As we have already discussed (eq. (13)) a possible gauge choice is to put to zero the supercoordinates η (eq. (13)). After such a gauge fixing the η–exponent drops out of $K(Z)$ in (23) and the resulting Cartan form components become polynomials of the 4-th power in θ.

For instance, the vector supervielbeins which form the induced worldvolume metric have the following structure (for simplicity we skip numerical coefficients)

$$E^i(x,r,y,\theta) = \left(\frac{r}{R}\right)^2 dx^i + iD\theta\Gamma^i\theta, \tag{24}$$

$$E^r = \frac{R}{r}dr + \left(\frac{r}{R}\right)^2 dx^i \theta\Gamma_i h^r \theta,$$

$$E^{a'}(x,r,y,\theta) = e^{a'}_{S^7}(y) + \left(\frac{r}{R}\right)^2 dx^i \theta\Gamma_i h^{a'} \theta,$$

$$D\theta^\alpha = d\theta^\alpha + E^A(\theta g_A)^\alpha + \left(\frac{r}{R}\right)^2 dx^i (\theta\Gamma_i h^A \theta)(\theta g_A)^\alpha,$$

where $E^A = e^A(x,r,y) + \left(\frac{r}{R}\right)^2 dx^i(\theta\Gamma_i h^A\theta)$ are supervielbeins and superconnections associated with the generators D, $P_{a'}$ and M_{ab} and $e^A(x,r,y)$ are their bosonic values at $\theta = 0$.

We see that supervielbeins and superconnections have a relatively simple form, though if we substitute them into the supermembrane action we will get an action for a superconformal field theory which still has a rather complicated structure of field interactions, which hinders the analysis of this theory.

Much simpler form (up to the second power in Θ) of the supervielbeins and superconnections might be obtained if in the exponent representation (23), instead of putting to zero η we might put to zero θ. Such a κ–symmetry gauge choice would be compatible with an (anti)static gauge of the worldvolume diffeomorphisms, when, for example, the worldvolume time is identified with minus AdS time coordinate

$$\xi^0 = -x^0, \quad \xi^1 = x^1, \quad \xi^2 = x^2. \tag{25}$$

Then we would have

$$E^i = (\frac{r}{R})^2(dx^i + id\eta\Gamma^i\eta), \quad E^r = e^r(x,r,y), \quad E^\alpha = \frac{r}{R}d\eta^\alpha, \tag{26}$$

which look very much like flat superspace covariant superforms.

This simple κ–symmetry gauge was proposed in [4] and [12], and is called a supersolvable algebra gauge, or a Killing spinor gauge. It is called supersolvable since when $\theta = 0$ the remaining generators in the parametrization of the coset element $K(Z)$ in (23) form a sub–superalgebra of $osp(8|4)$, which is an extension of a $d = 3$ super–Poincare algebra by the dilatation generator D.

And it is called a Killing spinor gauge since it corresponds to an appropriate choice of the solution of the AdS Killing spinor equation.

In this gauge the supermembrane action would take much simpler form, but here appears a problem that this gauge is not always admissible. For instance, it is not compatible with a natural static vacuum solution of the superbrane equations of motion when

$$\xi^i = x^i, \quad r = const, \quad y^{a'} = const, \quad \Theta = 0, \tag{27}$$

i.e. when the brane completely lives in a three–dimensional slice of the AdS_4 space–time. For such a solution $\eta = 0$ is an admissible gauge. Note that the static vacuum solution (27) is a BPS saturated state since it is invariant under the 16 standard supersymmetries Q_α and (spontaneously) breaks special superconformal symmetry S_α, which is nonlinearly realized on the excitations over this vacuum solution.

The $\theta = 0$ gauge would be compatible with an (anti) static configuration with the reverse orientation of time (or a space) coordinate, i.e.

$$\xi^0 = -x^0, \quad \xi^1 = x^1, \quad \xi^2 = x^2, \quad r = const, \quad y^{a'} = const, \quad \Theta = 0, \tag{28}$$

but this is not a solution of the superbrane equations, unless r is zero. And when $r = 0$ such a brane configuration shrinks to a point at the AdS horizon. Physically this configuration describes an antibrane which is attracted by a bunch of branes whose metric near horizon is close to that of $AdS \times S$.

There may exist, however, anti–static brane configurations, compatible with the Killing gauge, which extend along the radial coordinate of AdS and/or somehow nontrivially wind around compact directions of the sphere. In general, such brane configurations will break all supersymmetries of the superbackground. However, an action which describes fluctuations over these configurations will have a simple fermionic structure due to the simple form of the supervielbeins (26), and it would be of interest to study the properties of such theories.

The above examples teach us that the gauge choice is a subtle point and depends on which classical solution of field equations one deals with. Recently this problem has been also discussed in the case of a D0–brane in $AdS_2 \times S_2$ [13].

An example of the use of the Killing spinor gauge ($\theta = 0$) is a IIB superstring propagating in the $AdS_5 \times S^5$ superbackground. The $AdS_5 \times S^5$ superbackground can be viewed as a large N limit of coincident D3–branes. The Killing gauge is compatible with superstring κ–symmetry since the κ-symmetry projector now differs from the one used to impose the Killing gauge, the latter being related to the D3–brane κ–symmetry projector.

This theory has been considered by Pesando, Kallosh and Rahmfeld, and Kallosh and Tseytlin [2].

Using the $AdS_5 \times S^5$ supervielbeins in the Killing gauge, which have the form analogous to that written in (26), one can obtain the following superconformal action for the IIB superstring

$$S = -\frac{1}{2}\int d^2\xi \, \Big[\sqrt{-g}\, g^{ij}\Big(y^2(\partial_i x^p - 2i\bar\eta\Gamma^p\partial_i\eta)(\partial_j x^p - 2i\bar\eta\Gamma^p\partial_j\eta) + \frac{1}{y^2}\partial_i y^t \partial_j y^t\Big) \\ + 4i\epsilon^{ij}\partial_i y^t \bar\eta\Gamma^t\partial_j\eta\Big] \,, \qquad (29)$$

where now ξ^i $(i = 0,1)$ parametrize the superstring worldvolume, $g_{ij}(\xi)$ is an intrinsic (auxiliary) worldvolume metric, x^p $(p = 0,1,2,3)$ denote coordinates parallel to the D3–branes; $y^t = (r, y^{a'})$, which include the AdS radial coordinate and the S^5 coordinates, stand for the coordinates orthogonal to the D3–branes, and $y^2 = y^t y^t$.

This action differs from a IIB string action in a flat $D = 10$ superbackground, with κ–symmetry being gauge fixed in the same way as in (29), by the factors y^2 and $\frac{1}{y^2}$.

Now, having the action for a IIB superstring in $AdS_5 \times S^5$ one can study classical and quantum properties of this theory. First steps in this direction were undertaken by Kallosh and Tseytlin. It has been realized that it is not obvious that the superstring equations yielded by such an action admit supersymmetric classical solutions. It would be of interest to analyze a duality relation of this theory with a $D = 4$ Yang–Mills theory on the boundary of AdS. For this one should also know a corresponding D3–brane action in $AdS_5 \times S^5$ which would produce the Yang–Mills theory. This action is still under construction due to problems discussed above.

In conclusion what we have learned about the superbranes in the AdS backgrounds is that different gauge choices for fixing κ–symmetry of the original brane action may result in different worldvolume actions (and field theories). This is because we deal with topologically nontrivial backgrounds such as $AdS \times S$, and the most gauge fixing conditions are only locally admissible, and/or implicitly reflect how the brane is embedded into the background. The problem of finding globally defined conditions for gauge fixing brane actions in $AdS \times S$ has been considered in [14]. The light-cone gauge formalism for theories in AdS spaces has been developed in [15].

An unexpected observation which we have made is that actions which admit classical vacuum configurations preserving effective worldvolume supersymmetry have more complicated structure of the fermionic sector than actions for which the existence of supersymmetric brane configurations is problematic.

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